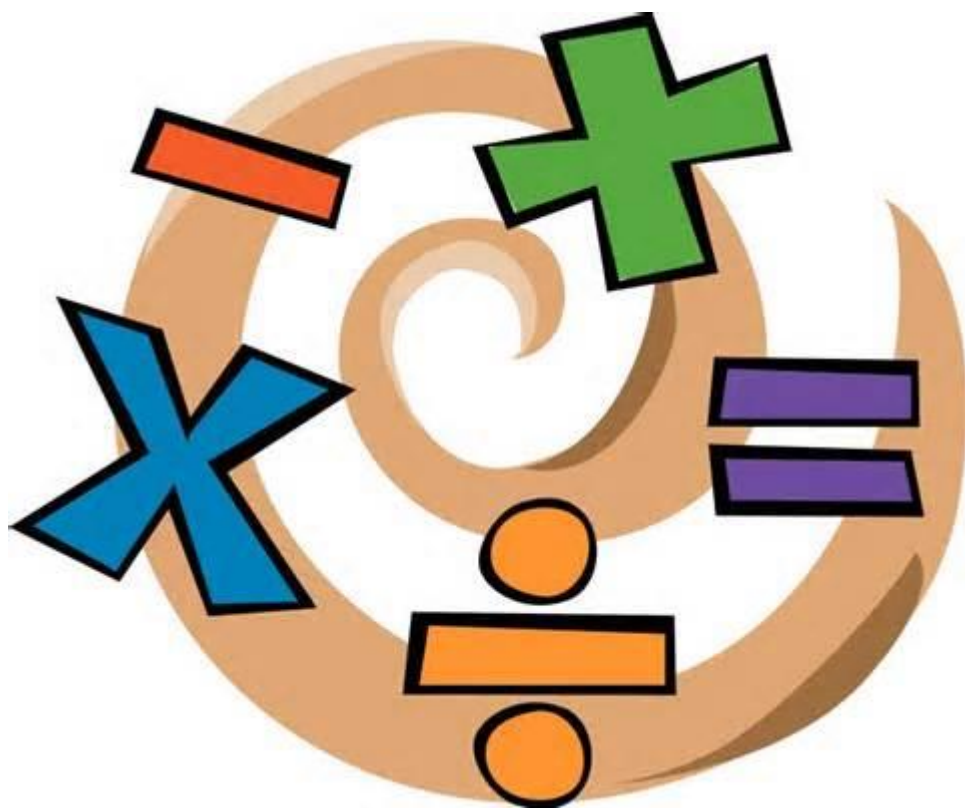


# Addition and Subtraction



Children will use a wide range of practical equipment to combine groups of objects to find a total. Practical resources will support children's development of mental pictures and images.

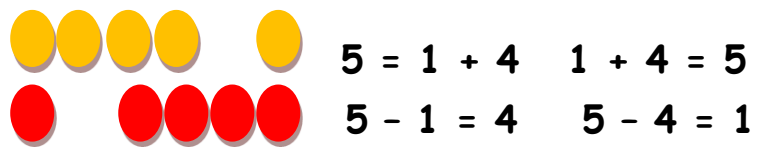
Children will begin to understand *commutativity* and the *principle of exchange*.

Children will be confident in using the terms 'worth' and 'value' when talking about single-digit numbers.

Children can represent calculations using objects and talk about their *representations*.



Fingers



Beads or any objects



Pegs



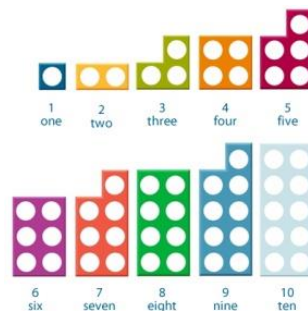
Cubes



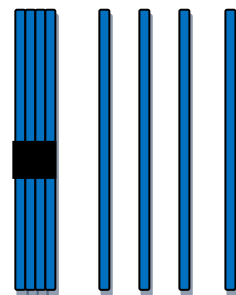
Counters



Cuisenaire Rods



Numicon



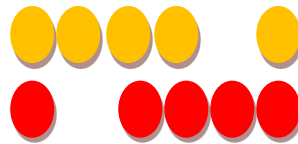
Straws

Children will use a wide range of practical equipment to physically remove an amount from the group to find the total remaining. Practical resources will support children's development of mental pictures and images.

Children can represent calculations using objects and talk about their *representations*.



Fingers



$$5 = 1 + 4 \quad 1 + 4 = 5$$

$$5 - 1 = 4 \quad 5 - 4 = 1$$

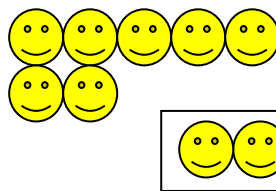
Beads or any objects



$$5 - 3 = 2$$



Pegs



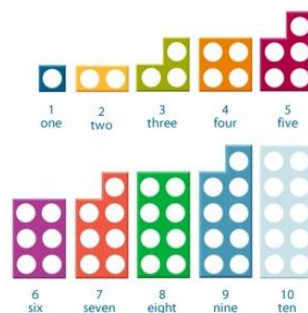
Counters



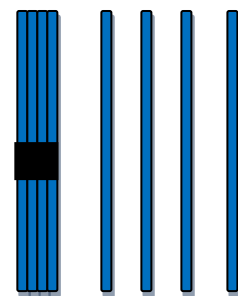
Cubes



Cuisenaire Rods



Numicon



Straws

Children will also be introduced to the language of comparison including equal use of the vocabulary 'less' and 'more'



There are more red cubes than blue.

There are less blue cubes than red.

# Stage 2

# Addition



Practical resources will support children's development of mental pictures and images. As children become secure with these, they will begin to develop ways to represent their mental images and their practical resources using pictures.

Children will begin to use number sentences alongside their pictures and practical resources.

Children will begin to think and talk flexibly about addition.

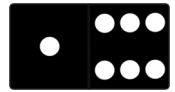
The direct link between addition and subtraction should be made explicit when using models and representations.



$$2 + 3 = 5$$



$$10 = 5 + 5$$



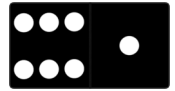
$$1 + 6 = 7$$



$$5 = 3 + 2$$



$$10 = 7 + 3$$



$$6 + 1 = 7$$



$$2 + 3 = 5$$

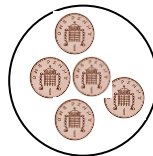


$$10 = 3 + 7$$

9 and 1 more is 10

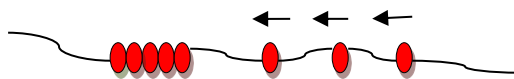
9 add 1 equals 10

$$9 + 1 = 10$$



5 and 1 more is?

6



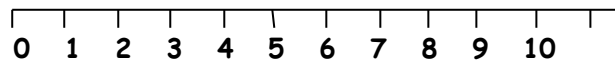
6 7 8

5 and 2 more is?

6,7

5 and 3 more is?

6,7,8

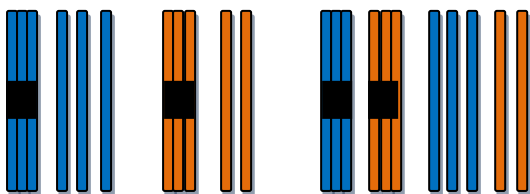


Count on one, two or three

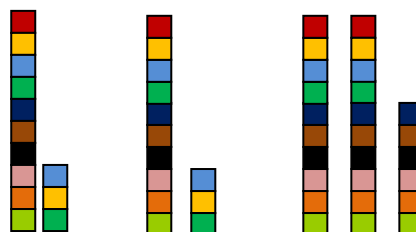
$$8 + 2 = 10$$

$$10 = 6 + 4$$

$$4 + 6 = 10$$



$$13 + 12 = 25$$



$$13 + 12 = 25$$

Practical resources will support children's development of mental pictures and images. As children become secure with these, they will begin to develop ways to represent their mental images and their practical resources using pictures.

Children will begin to use number sentences alongside their pictures and practical resources.

Children will begin to think and talk flexibly about subtraction and makes links to the inverse of addition.

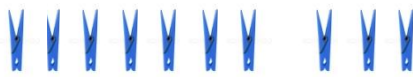
Children will gain an understanding that subtraction is not commutative and so the numbers in a calculation can be in any order but will result in a different answer.

The direct link between addition and subtraction should be made explicit when using models and representations.



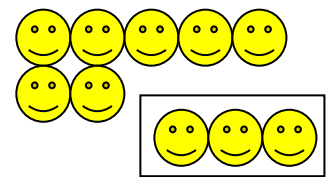
$$6 + ? = 10 \quad ? + 6 = 10$$

$$10 - 6 = ? \quad 10 - 4 = 6$$



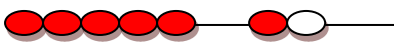
$$10 - 7 = 3$$

$$10 - 3 = 7$$



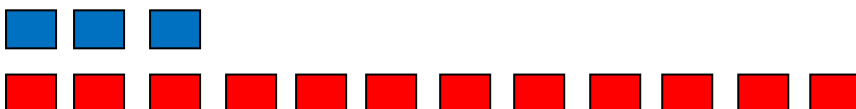
$$10 - 7 = 3$$

$$10 - 3 = 7$$



$$6 - 2 = 4$$

Children will continue to be introduced to the language of comparison and link it to finding the difference structure or subtraction.



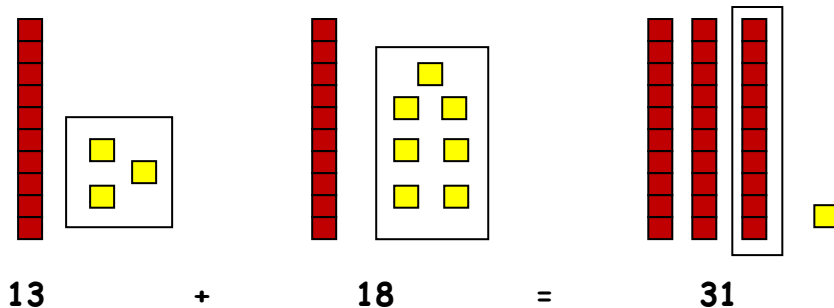
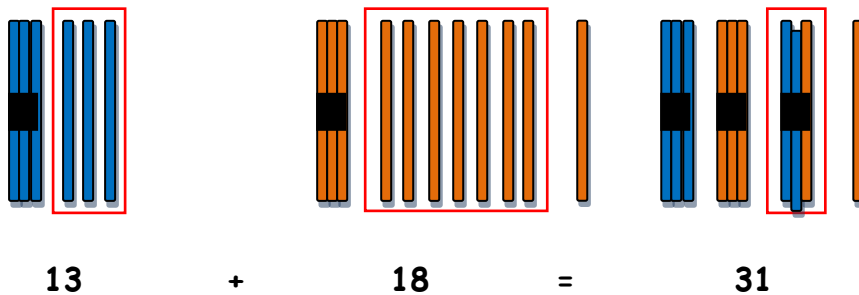
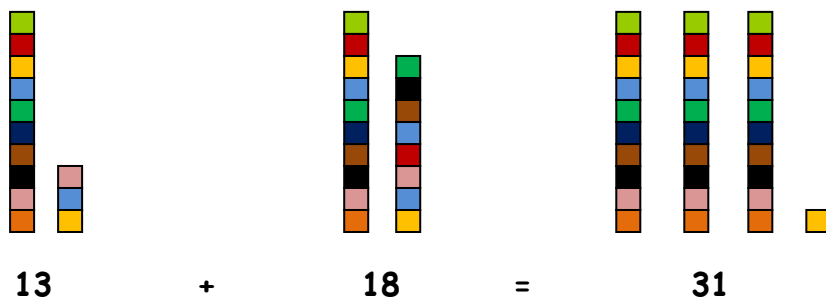
- There are more red than blue
- There are less blue than red
- There are 9 more red than blue
- There are 9 less blue than red

Children will now be confident in using concrete practical equipment to help them combine groups of objects with numbers up to 20.

They will continue to use practical equipment as they begin to also use *number tracks*, *number lines* and *hundred squares* to support their mental methods.

Children will start to work with totals greater than 20 which require them to apply their knowledge of the *principle of exchange*. They will talk confidently about this.

$13 + 18$



# Stage 3

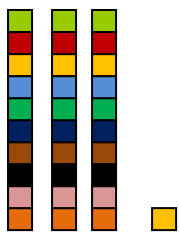
# Subtraction

Children will now be confident in using concrete equipment to help them 'take away' and 'find the difference'.

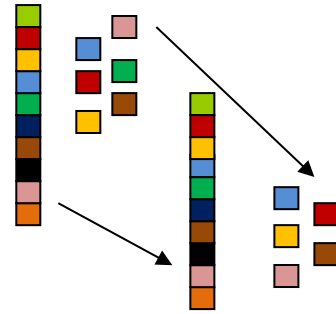
Children will continue using practical equipment as they begin to also use number tracks, number lines and hundred squares to support their mental methods.

Children will start to work with numbers greater than 20 which require them to apply their knowledge of the principle of exchange and will talk confidently about this.

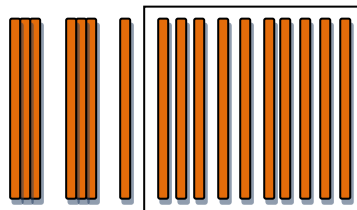
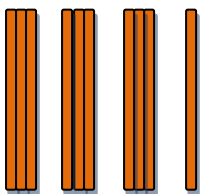
$31 - 15$



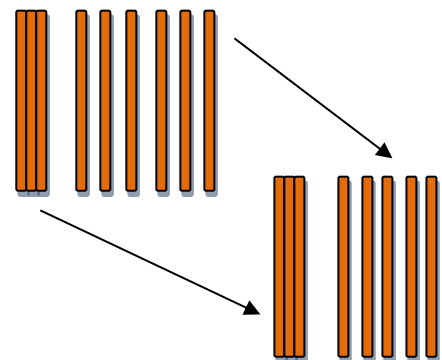
31 is repartitioned into 20 and 11 using the principle of exchange in order to enable us to remove the five ones associated with 15



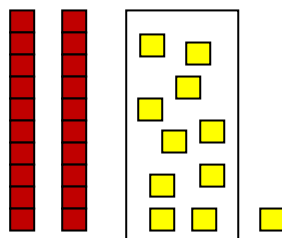
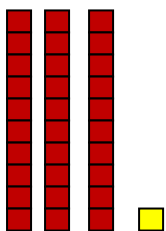
15 can now be removed from the 31 leaving 16



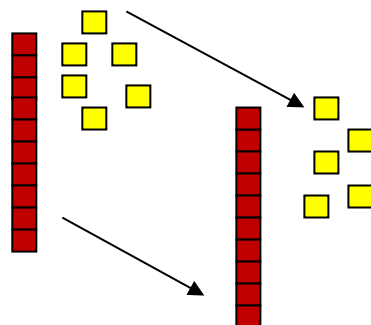
31 is repartitioned into 20 and 11 using the principle of exchange in order to enable us to remove the five ones associated with 15



15 can now be removed from the 31 leaving 16



31 is repartitioned into 20 and 11 using the principle of exchange in order to enable us to remove the five ones associated with 15



15 can now be removed from the 31 leaving 16

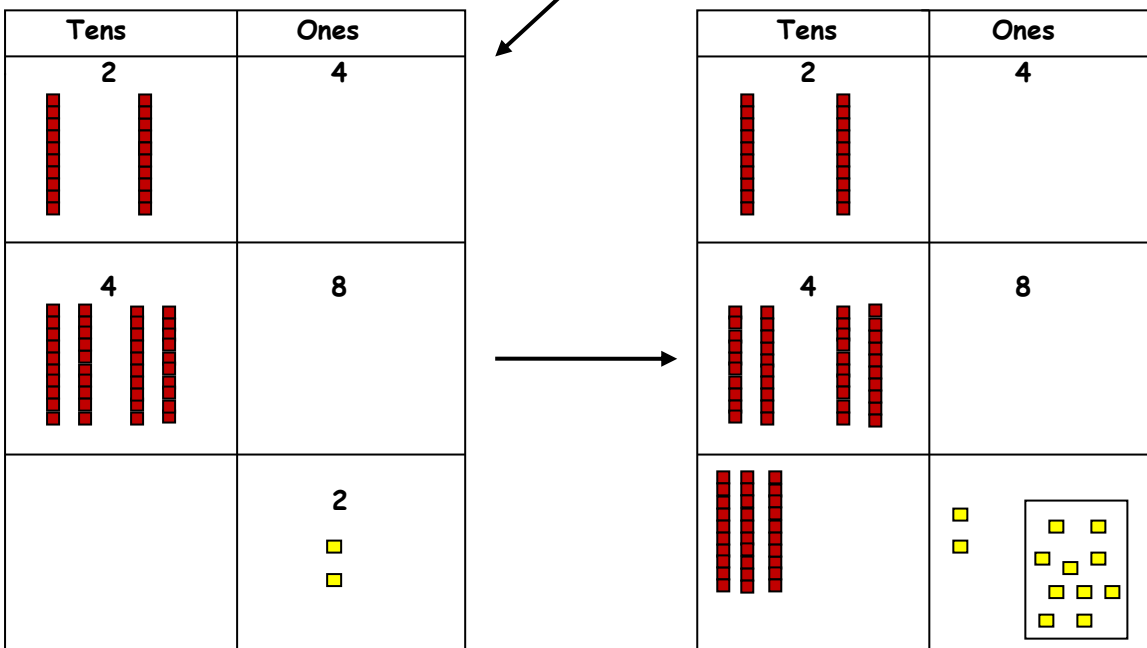
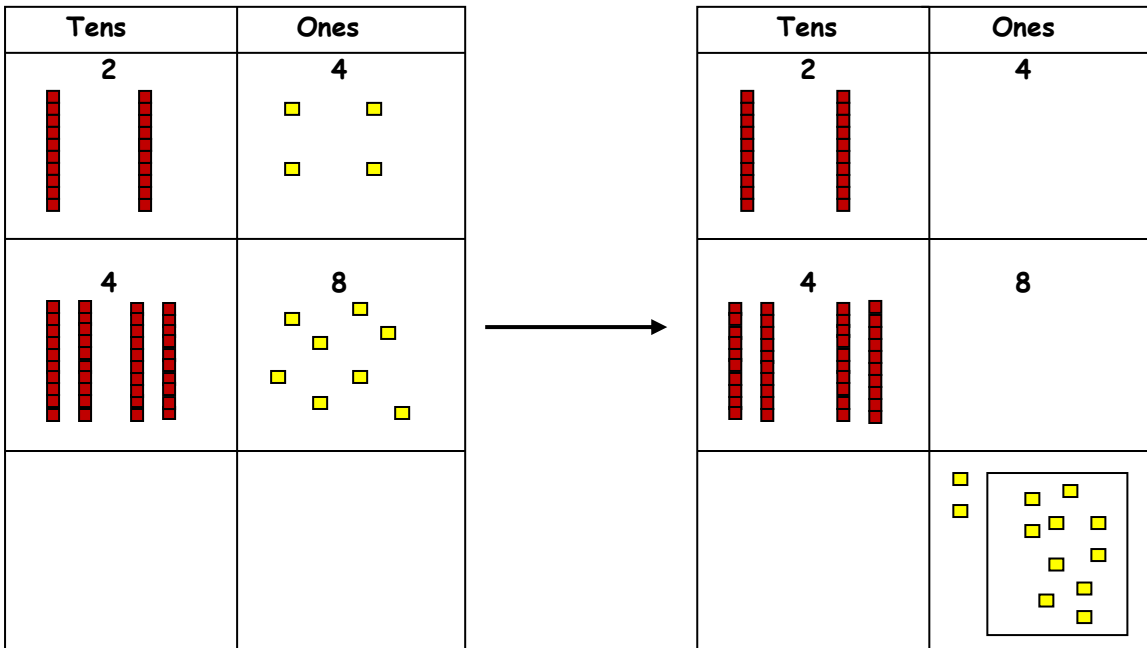
As children become accustomed to repartitioning numbers, they can be introduced to formal notation of the repartitioning.

2	1
<del>3</del>	<del>1</del>
This is now 20 and 11	

Children are now confident in using concrete equipment to combine objects using the principle of exchange.

They will now begin to organise their concrete equipment e.g. Dienes, straws, place value counters, in a vertical manner where their combined totals are situated at the bottom.

$24 + 48$



1

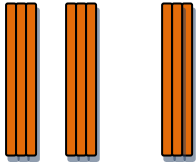



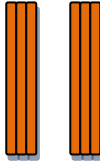
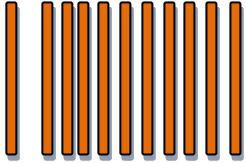
12 ones exchanged to 1 ten and 2 ones


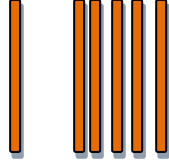
Children are now confident in using concrete practical equipment to 'take away' and 'find the difference' using the principle of exchange appropriately.


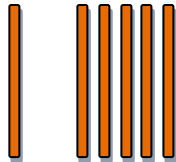
Children will now begin to organise their concrete equipment e.g. Dienes, straws, Place value counters) in a vertical manner where the amount that remains at the end of the calculation is situated at the bottom.

$$31 - 15$$

Tens	Ones
3 	1 
1	5

Tens	Ones
2 <del>3</del> 	1 1 
1	5

Tens	Ones
2 <del>3</del> 	1 1 
1	5

Tens	Ones
2 <del>3</del>	1 1
1	5
	

14 can now be removed from the 31

The remaining equipment can then be slid down to the answer box showing what is left.

Children are now be secure in organising their concrete equipment in a vertical manner where their combined totals are situated at the bottom.

They will be now be able to make the links between this representation and the formal column addition when seen alongside each other.

$23 + 48$

Tens	Ones
2 	4 
4 	8 

$$\begin{array}{r} 24 \\ + 48 \\ \hline \end{array}$$

Tens	Ones
2 	4 
4 	8 
	11 

$$\begin{array}{r} 24 \\ + 48 \\ \hline \end{array}$$

11 ones  
exchanged  
to  
1 ten and  
1 ones

$$\begin{array}{r} 24 \\ + 48 \\ \hline 72 \\ 1 \end{array}$$

Tens	Ones
2 	4 
4 	8 
7 	2 

$$\begin{array}{r} 24 \\ + 48 \\ \hline 72 \\ 1 \end{array}$$

$$\begin{array}{r} 24 \\ + 48 \\ \hline 72 \\ 1 \end{array}$$

# Stage 5

# Subtraction



Children will now be secure in organising their concrete practical equipment in a vertical manner for subtraction using the principle of exchange appropriately.

Children will now be able to make the links between this representation and the formal column subtraction when seen alongside each other.

$$32 - 15$$

Tens	Ones
3 10 10 10	2 1 1
<hr style="width: 100%;"/> 1	<hr style="width: 100%;"/> 5
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>

Tens	Ones
<del>3</del> 2 10 10	<del>2</del> 3 1 1 1 1 1 1 1 1
<hr style="width: 100%;"/> 1	<hr style="width: 100%;"/> 5
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>

32 is repartitioned into 20 and 12 using the principle of exchange in order to enable us to remove the five ones associated with 15

Tens	Ones
<del>2</del> <del>3</del> 10	<del>1</del> <del>2</del> 1 1 1 1 1 1 1 1
<hr style="width: 100%;"/> 1	<hr style="width: 100%;"/> 5
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>

Tens	Ones
<del>2</del> <del>3</del>	<del>1</del> <del>2</del>
<hr style="width: 100%;"/> 1	<hr style="width: 100%;"/> 5
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>

15 can now be removed from the 31

Tens	Ones
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>

Tens	Ones
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>

The remaining equipment can then be slid down to the answer box showing what is left

Tens	Ones
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>

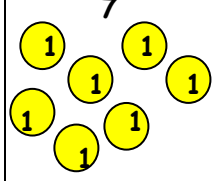
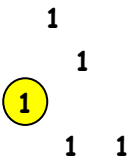
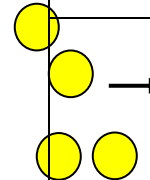
Tens	Ones
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>



15 can now be removed from the 31

32 is repartitioned into 20 and 12 using the principle of exchange in order to enable us to remove the five ones associated with 15

The remaining equipment can then be slid down to the answer box showing what is left



Children will have a full understanding of the links between concrete representation for column addition and the formal written method.

They will now be able to explore calculating with larger numbers using their understanding of the formal written method.

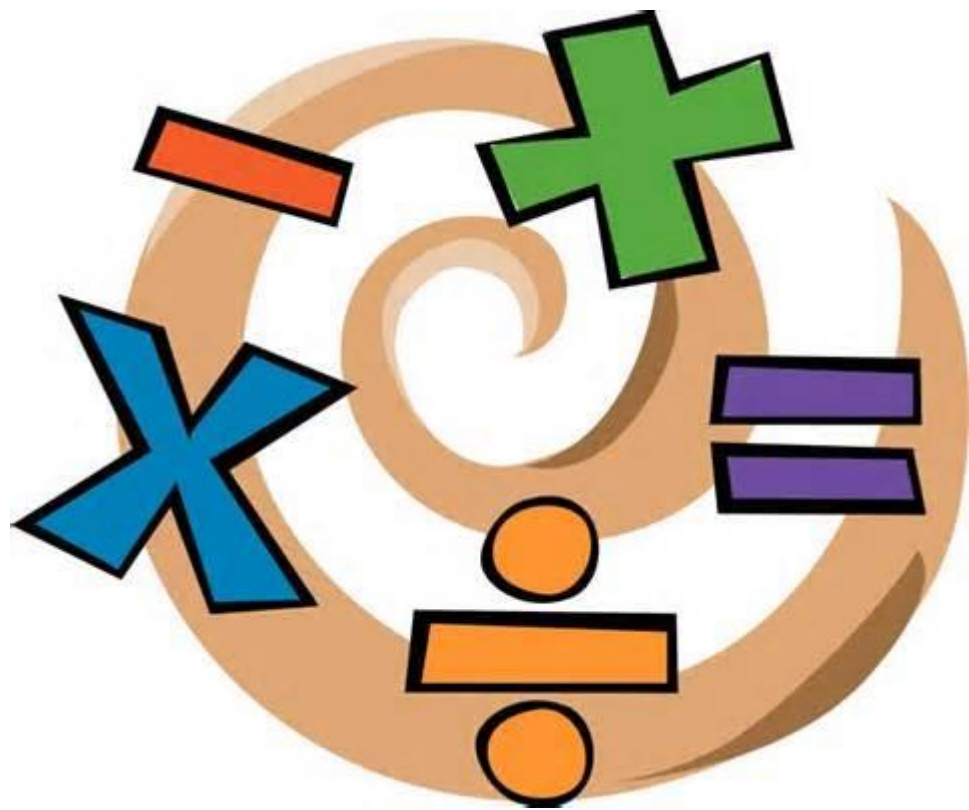
$$\begin{array}{r} 136 \\ + 285 \\ \hline 411 \\ \hline 11 \end{array}$$

#### Calculating with Decimals

When working with Decimals, the above stages should always be followed. This will allow children to develop their conceptual understanding. The use of concrete equipment is essential at these stages to secure understanding of the value of each digit in a number e.g. place value counters, money etc. Wherever possible, decimal calculations should be linked to real life experiences.



# Multiplication and Division

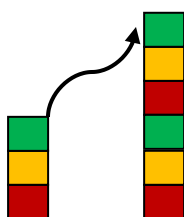
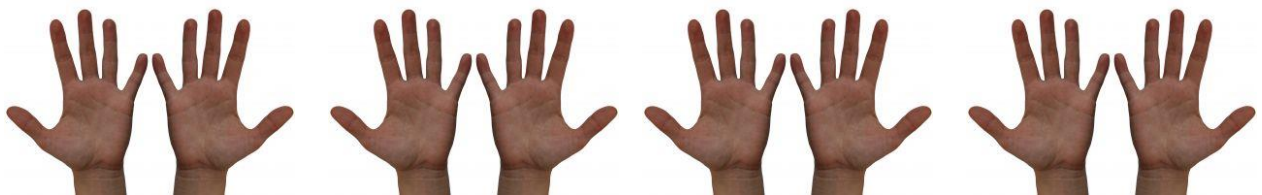
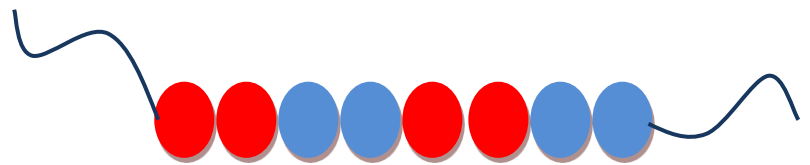


Children will use a wide range of practical equipment to experience practical opportunities involving equal sets or groups. Practical resources will support children's development of mental pictures and images.

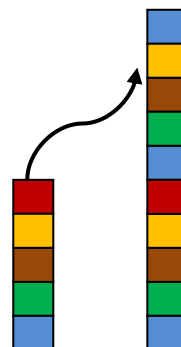
Children will begin to orally count in different multiples including twos, fives and tens making links to natural groupings. (E.g. legs on animals, pairs of shoes) and the practical resources used.

Children can begin to recognise and continue patterns of multiples using a range of practical resources e.g. building towers with cubes with two of each colour)

They will begin to use the language and associated representations of doubling.



Double 3 is 6



Double 5 is 10

# Stage 1

# Division

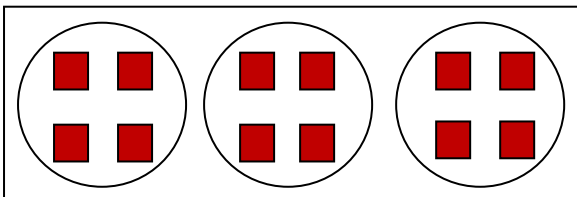
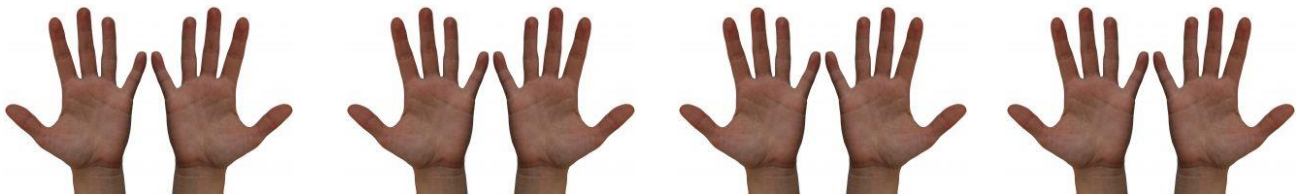
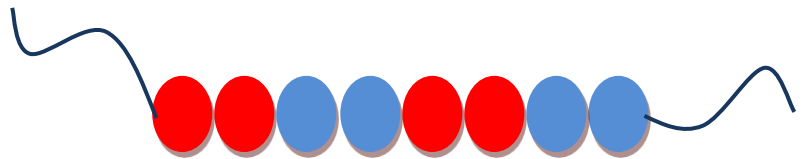


Children will explore the language of sharing. They will use a wide range of practical equipment to experience practical activities in 'sharing' objects between a small number of groups/people. The emphasis being on sharing equally.

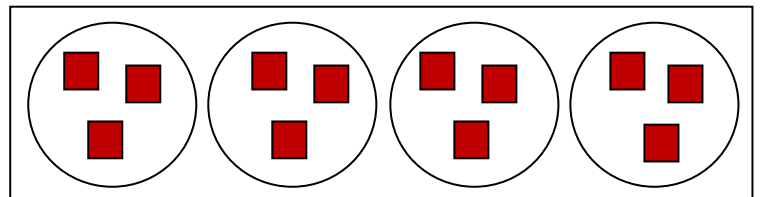
Working alongside this, children should be introduced to 'grouping' objects as a **representation** of division, (E.g. each person in the group gets 2), with the emphasis on equal groups. 'Sharing' and 'grouping' should have equal weighting when carrying out these activities.

Children will begin to use the language and associated **representation** of halving.

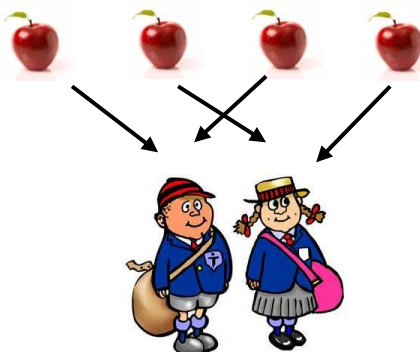
Children can be encouraged to develop ways of recording their findings using pictures.



12 shared into 3 equal groups  
12 shared equally into groups of 4



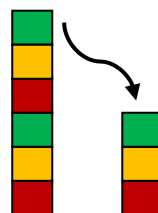
12 shared into 4 equal groups  
12 shared equally into groups of 3



4 apples shared between 2 people

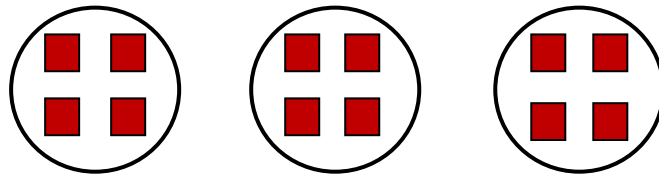


4 apples, how many people can have 2 each?

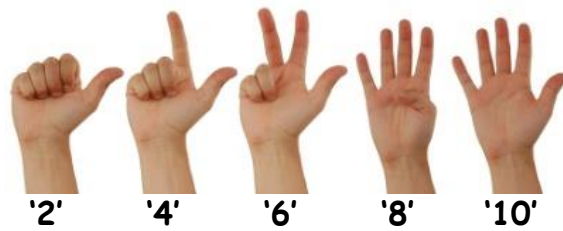


Half of 6 is 3

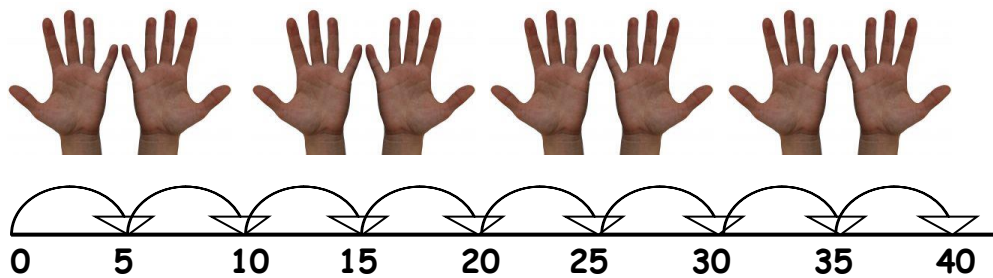
Children will begin to arrange objects into equal groups to support counting.



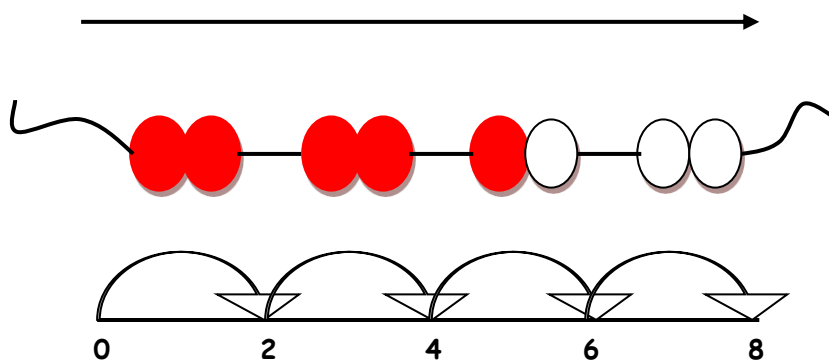
Children will continue to count in multiples and begin to relate this to multiplication through finger counting.



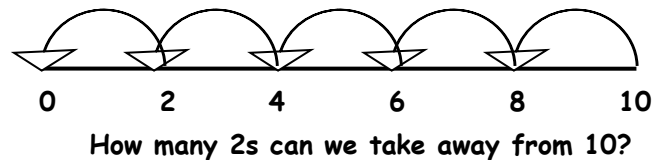
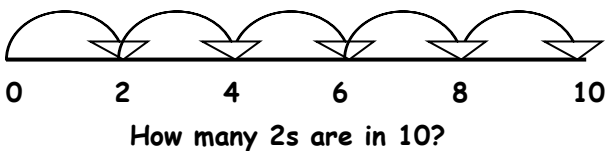
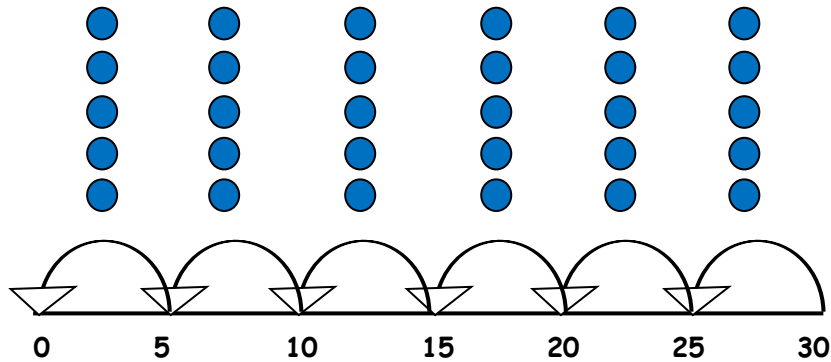
Children will be introduced to a variety of **representations** of repeated addition. They will see the **representations** alongside each other and begin to make connections between them.



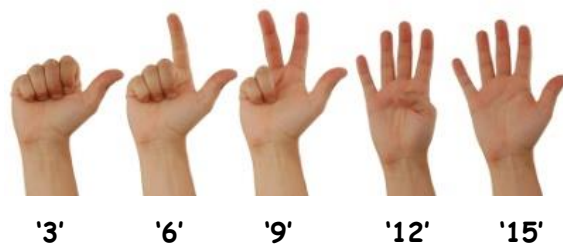
Count in twos from zero



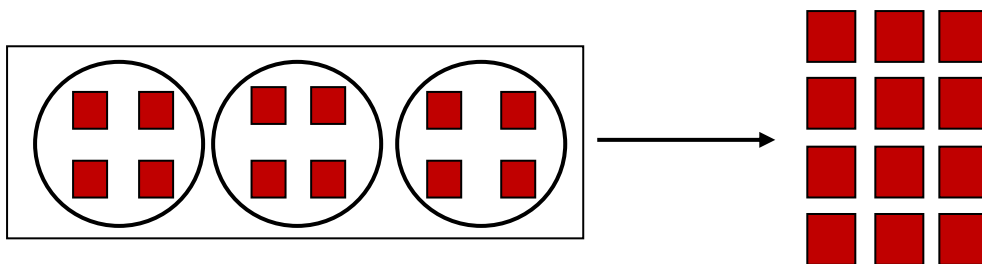
Children will relate the grouping of objects to repeated subtraction and begin to represent this using a number line. They will do this using concrete equipment.



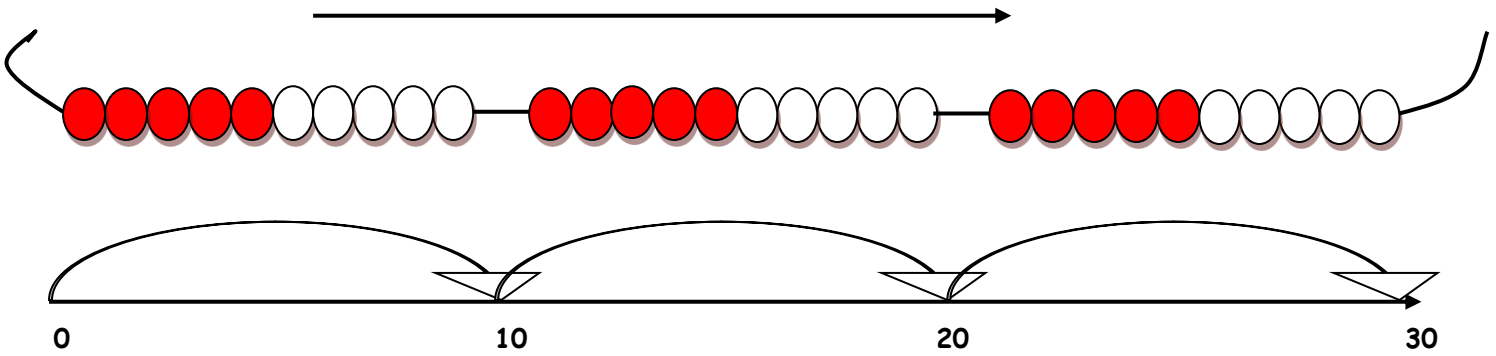
Children will use their knowledge of counting up in multiples to solve division calculations and recognise that this is the *inverse of multiplication*.



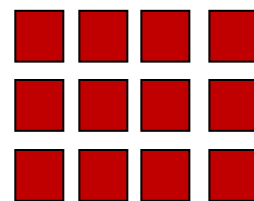
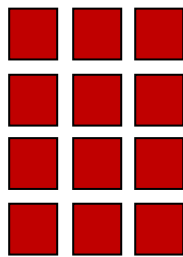
Children will continue to group and share equally using concrete equipment. They will begin to organise their groups into an array rather than scattered groupings.



Count in tens from zero



Children will be introduced to the array for small numbers as a way of organising groups to show repeated addition and **commutativity**. They will do this using concrete equipment. They should explore arrays using equipment from the world around us e.g. egg boxes, cake cases in baking trays. They should use them to answer questions such as 'How many eggs would we need to fill an egg box?' 'How do you know?'





The direct link between multiplication and division should be made explicit when using representations and models.

Children will continue to make links between division and fractions. They will be aware that the division sign is the equivalent to the fraction line and so  $s \div r$  can be written as

$$\frac{s}{r}$$

$$1 \div 2$$

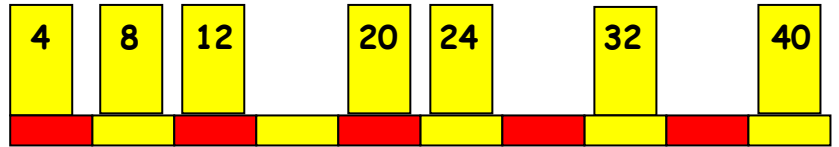
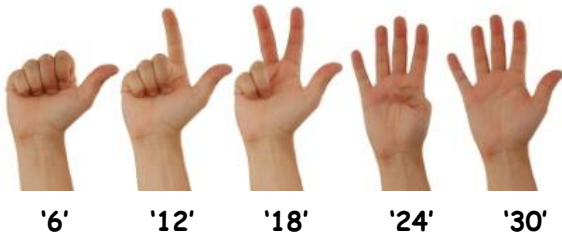
A fraction representation of 1 divided by 2. The numerator '1' is inside a blue circle, and the denominator '2' is inside another blue circle. A thick blue horizontal line is positioned between the two circles, serving as the fraction bar.

# Stage 3

# Multiplication

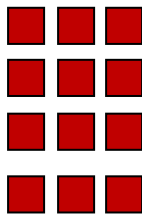


Children will continue to count in multiples and relate this to multiplication through finger counting.

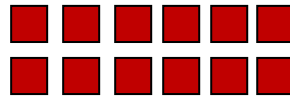


Children will be able to model a calculation using a practical array which shows an effective method of counting and the link to repeated addition. Children need to explore related multiplication facts of a given number by making a variety of arrays and explaining what they show.

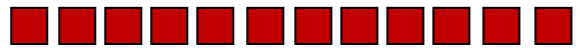
Representing  
12



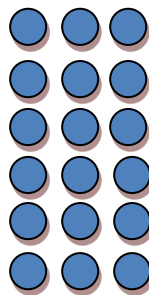
$$3 \times 4 = 12$$
$$4 \times 3 = 12$$



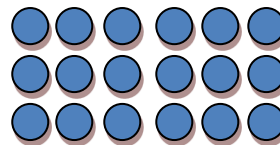
$$2 \times 6 = 12$$
$$6 \times 2 = 12$$



$$1 \times 12 = 12$$
$$12 \times 1 = 12$$



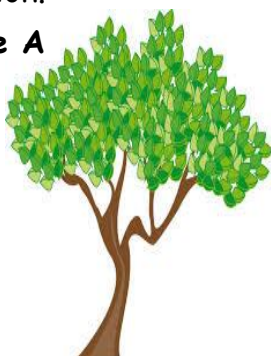
$$3 + 3 + 3 + 3 + 3 + 3 = 18$$



$$6 + 6 + 6 = 18$$

The children should be confident with their use of the language of scaling when talking about multiplication.

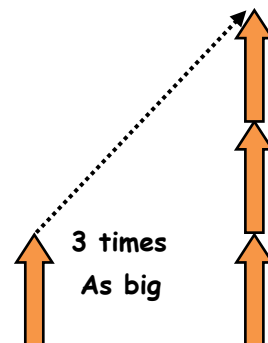
Tree A



↑  
Tree A is  
3 times as tall  
As Tree B



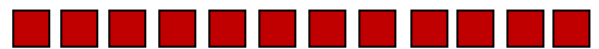
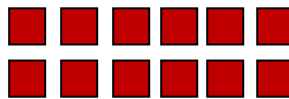
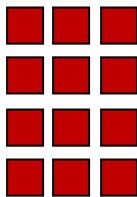
Tree B



Children will use their knowledge of counting in multiples to support the inverse of multiplication and repeated subtraction.

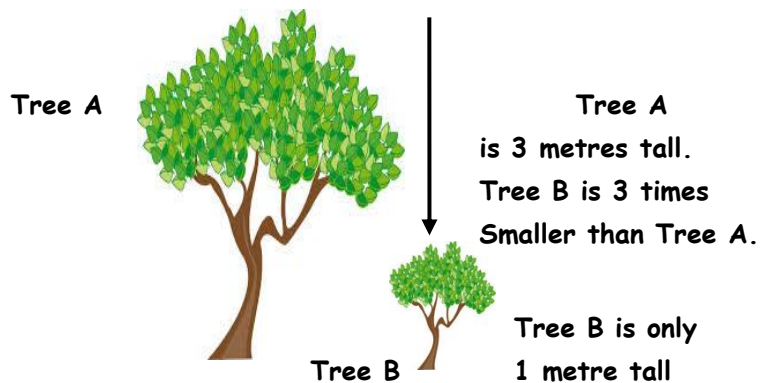
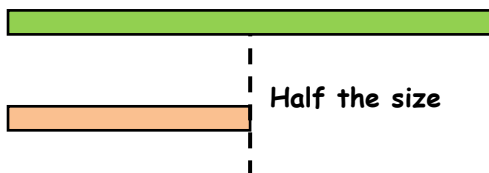
Children will build on their use of concrete arrays for division recognising the links to repeated subtraction and the inverse of multiplication in order to derive the associated division facts. Children need to explore related division facts of a given number by making a variety of arrays and explaining what they show.

Representing  
12



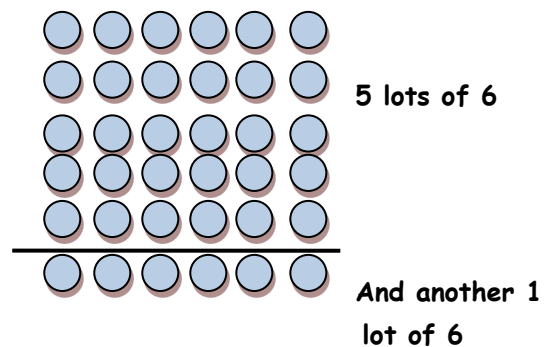
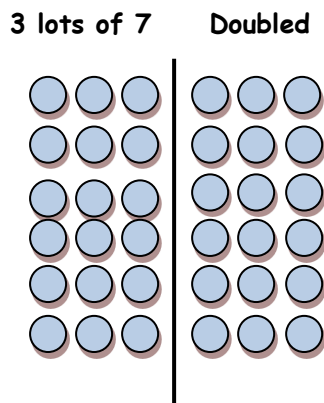
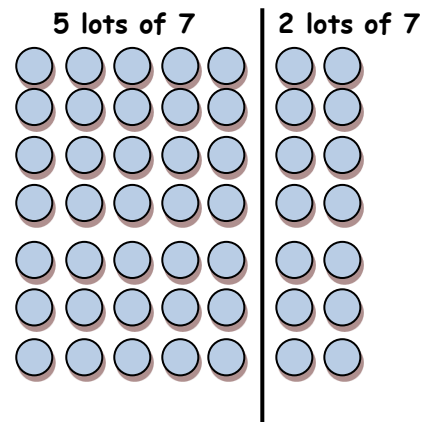
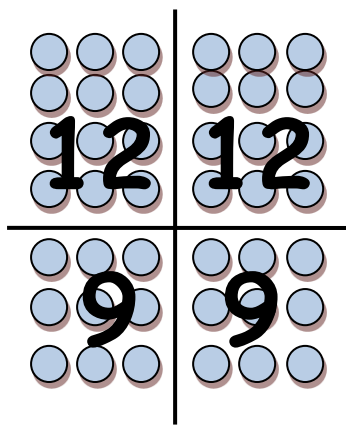
12 into \_\_\_\_\_ equal groups gives \_\_\_\_\_ in each group  
12 into equal groups of \_\_\_\_\_ gives \_\_\_\_\_ groups.

Children should be confident with their use of the language of scaling when talking about division with links made to simple fractions (e.g. half the size, three times smaller).



Children will explore practical arrays for larger numbers. They will think flexibly when working with arrays. They will be encouraged to look at arrays beyond repeated addition. They will look for 'friendly' numbers to help them efficiently calculate totals within arrays. E.g. for  $6 \times 7$ ... children may find counting in 6s and 7s tricky so they can look for 'friendly' numbers which are easier to calculate e.g.  $3 \times 3$ ,  $4 \times 3$ ,  $3 \times 3$ ,  $4 \times 3$

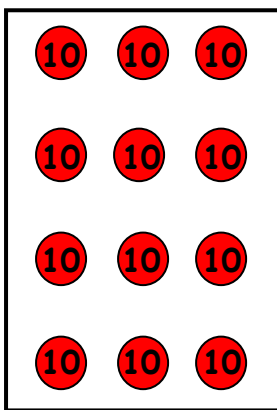
## Thinking flexibly about $6 \times 7$



Children should continue to experience the language of scaling e.g. scaling up pictures by powers of 10, multiplying by powers of 1000 in converting between units of measures

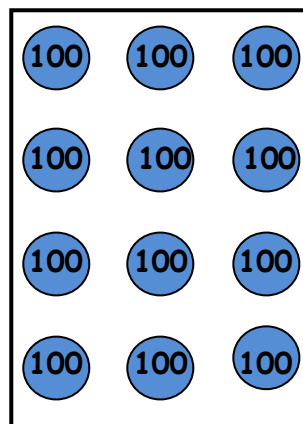
Children will continue to organise groups into an array now working with larger numbers by either grouping or sharing. Children will be able to explain all the facts they know about a given array with no remainder. Children should be making arrays with the equipment to establish 'How many in each group?' or 'How many groups?' Children should continue to experience the language of scaling e.g. scaling down pictures by dividing by powers of 10, dividing by powers of 1000 in converting between units of measure.

$$120 \div 4$$



120 shared equally between 4 is 30  
 120 shared equally between 3 is 40  
 4 equal groups of 30 make 120  
 3 equal groups of 40 make 120

$$1200 \div 4$$



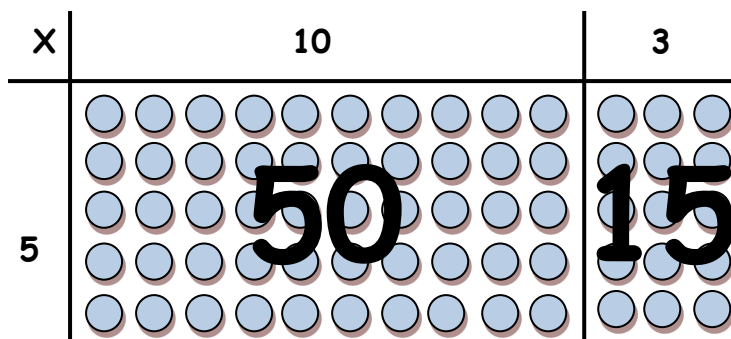
1200 shared equally between 4 is 300  
 1200 shared equally between 3 is 400  
 4 equal groups of 300 make 1200  
 3 equal groups of 400 make 1200

Children will continue to explore larger numbers, using arrays. This will lead to the grid method of multiplication. Use of practical equipment may still be required for some children as they enter this stage.

At first children should see the array with the grid lines. When secure with this, children should move to using the grid method, displaying the numbers only.

Children should begin using the grid method for 2 and 3 digit numbers by 1 digit numbers. They should be given the opportunity to relate this to facts they know about arrays where needed.

Throughout this stage, children should be encouraged to *estimate* an approximate answer in order to check for reasonableness. This should become standard practice.



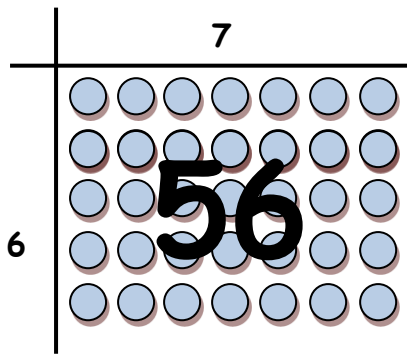
$$(5 \times 10) + (5 \times 3)$$

$$\begin{array}{r} 50 + 15 \\ 65 \end{array}$$

X	10	3
5	50	15

Children will continue to work with concrete arrays. They will explore known multiplication/division facts, with the use of grid lines, to begin to make the link to short division where numbers are easily divisible.

Children understand that the array within short division can be interpreted for both sharing between or equal groups, where the dots within the array represent 1.



How many equal groups of 6 can I make?

(grouping is represented in the columns)

or

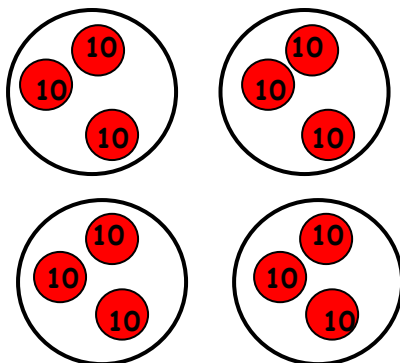
If these are put into 6 equal groups, how many in each group?

(sharing between is represented in the rows)

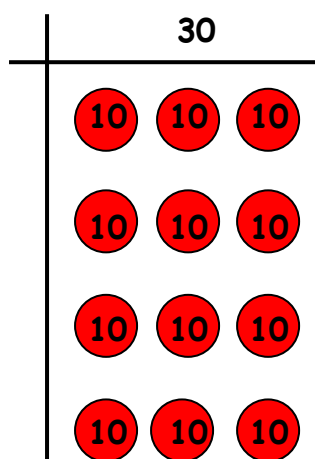
Children will begin to use counters within an array to show the sharing model of division. They will use their knowledge of the Principle of Exchange where necessary. At this stage, children are encouraged to consider the links between the sharing model and fractions.



120 can be exchanged for 12 tens in order to make an array



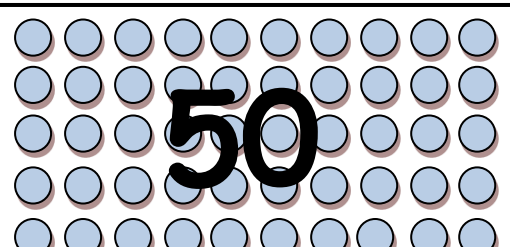
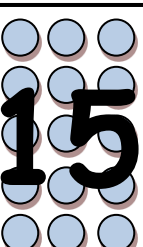
120 shared into 4 equal groups gives 30 in each group



It is explicit to see four times; 4 rows of 30, a  $\frac{1}{4}$  of 120 is 30.

We can divide the array into four parts and there is 30 in each part.

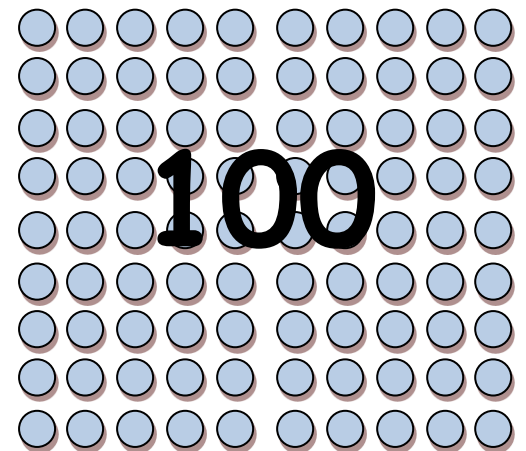
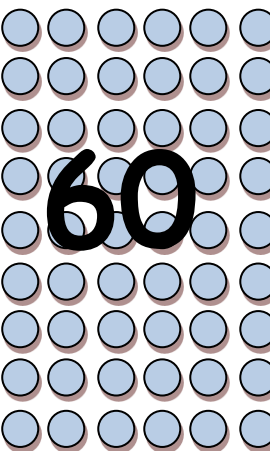
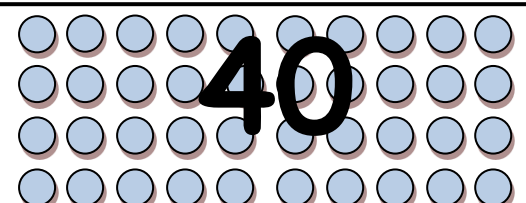
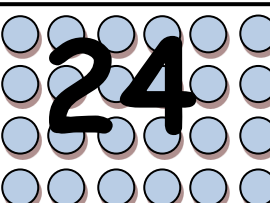
Children will now be secure in using the grid method for multiplying by 1 digit numbers. They will begin to explore the links between the grid method and the expanded method of short multiplication.

X	10	3
5	 50	 15

→

13
X 5
15 (5 x 3)
+ 50 (5 x 10)
65

Children will also begin to explore the use of arrays and the grid method for multiplying by 2 digit numbers.

X	10	6
10	 100	 60
4	 40	 24

X	10	6
10	100	60
4	40	24

$$\begin{array}{r}
 160 \\
 + 64 \\
 \hline
 224 \\
 \hline
 1
 \end{array}$$

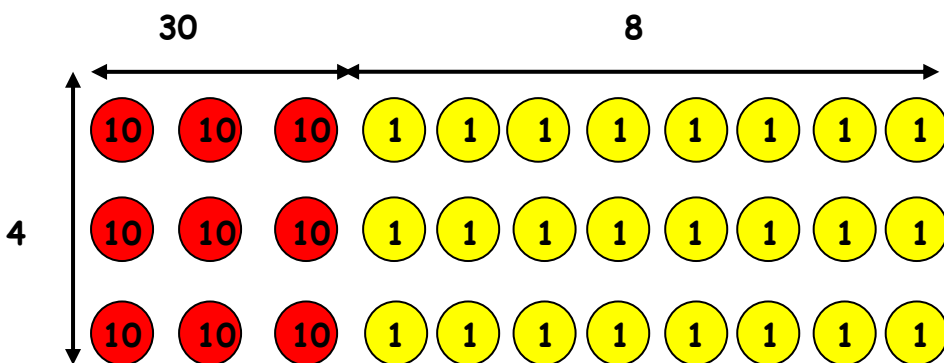
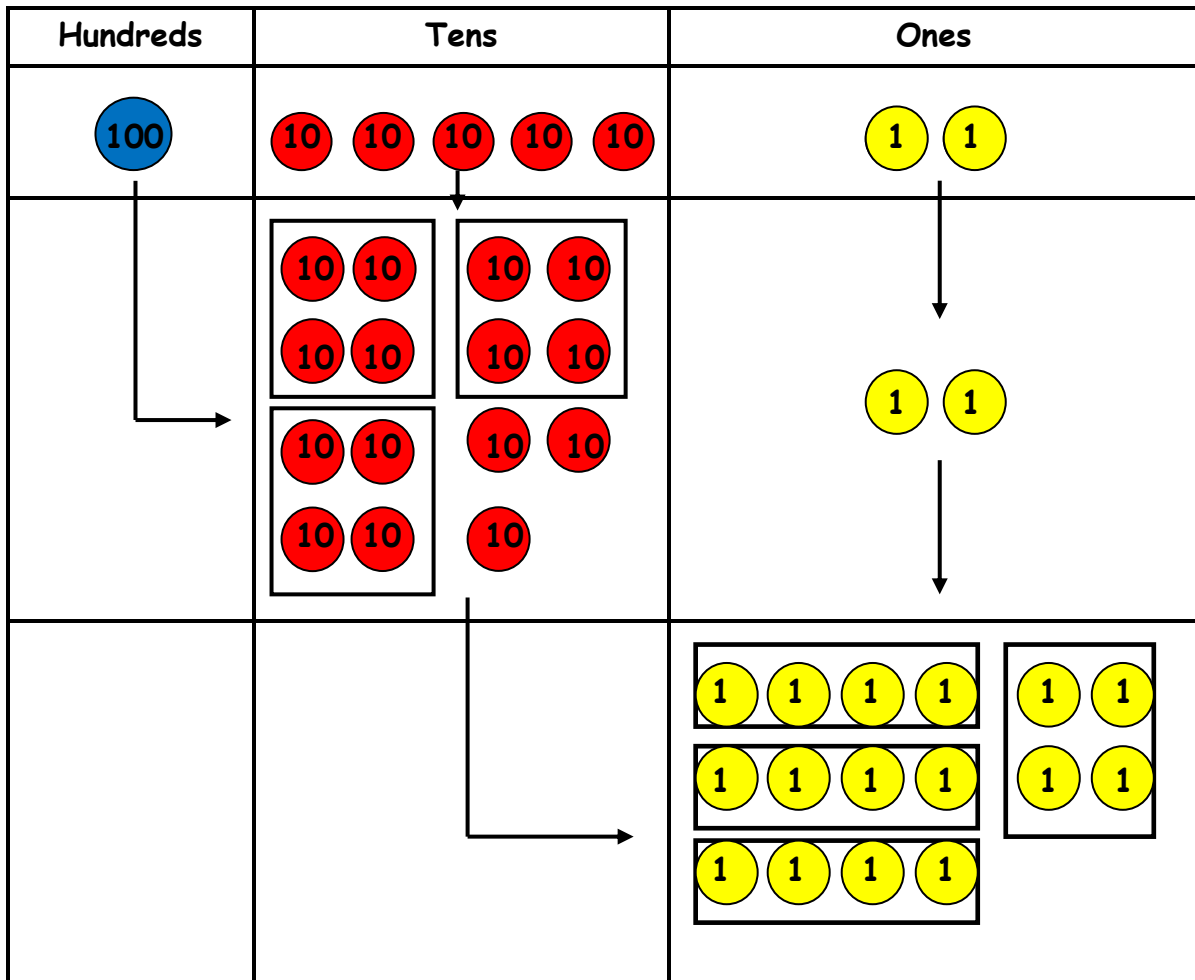


Children will work with equipment to divide any integer by a single digit divisor using their secure knowledge of the principle of exchange.

They will begin to be introduced to numbers that have remainders and will recognise and be able to talk about these when they do not 'fit' into the array.

Children will be introduced to the notation of short division, linking with the principle of exchange and how this relates to the practical representations.

Children continue to be encouraged to consider the links between the sharing model and fractions.



$$\begin{array}{r}
 38 \\
 4 \overline{) 152} \\
 \underline{12} \phantom{0} \\
 32 \\
 \underline{32} \\
 0
 \end{array}$$

In the array, it is explicit to see 38 four times; 4 rows of 38. This is the sharing model.

$\frac{1}{4}$  of 152 is 38.

We can divide the array up into 4 equal parts and there is 38 in each part.

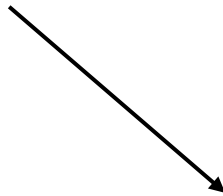




Children will now have a good understanding of the short multiplication method.

Children will now have an understanding of the expanded long multiplication method. They will begin to represent this as compact long multiplication.

$$\begin{array}{r}
 \phantom{X} 16 \\
 X 13 \\
 \hline
 18 \quad (3 \times 6) \\
 30 \quad (3 \times 10) \\
 60 \quad (10 \times 6) \\
 100 \quad (10 \times 10) \\
 \hline
 208 \\
 1
 \end{array}$$



$$\begin{array}{r}
 \phantom{X} 16 \\
 X 13 \\
 \hline
 48 \\
 160 \\
 \hline
 208 \\
 1
 \end{array}$$

### Calculating with Decimals

When working with Decimals, the above stages should always be followed. This will allow children to develop their conceptual understanding. The use of concrete equipment is essential at these stages to secure understanding of the value of each digit in a number e.g. place value counters, money etc. Wherever possible, decimal calculations should be linked to real life experiences.



Children will now be secure in using short division for one digit divisors and long division for two digit divisors with and integer quotient.

They will now explore the use of long division for two digit divisors which may include a remainder.

The children will begin to interpret remainders as whole number remainders, fractions or by rounding, as appropriate for the context.

$$\begin{aligned}
 1 \times 15 &= 15 \\
 2 \times 15 &= 30 \\
 4 \times 15 &= 60 & 20 \times 15 &= 300 \\
 8 \times 15 &= 120 \\
 10 \times 15 &= 150
 \end{aligned}$$

$$\begin{array}{r}
 \phantom{15} \overline{) 432} \text{ r } 12 \\
 - 300 \text{ (} 20 \times 15 \text{)} \\
 \hline
 132 \\
 - 120 \text{ (} 8 \times 15 \text{)} \\
 \hline
 12
 \end{array}$$

$$\begin{array}{r}
 \phantom{15} \overline{) 432} \frac{12}{15} \\
 - 300 \text{ (} 20 \times 15 \text{)} \\
 \hline
 132 \\
 - 120 \text{ (} 8 \times 15 \text{)} \\
 \hline
 12
 \end{array}$$

$$\begin{array}{r}
 \phantom{15} \overline{) 432} \frac{4}{5} \\
 - 300 \text{ (} 20 \times 15 \text{)} \\
 \hline
 132 \\
 - 120 \text{ (} 8 \times 15 \text{)} \\
 \hline
 12
 \end{array}$$

### Calculating with Decimals

When working with Decimals, the above stages should always be followed. This will allow children to develop their conceptual understanding. The use of concrete equipment is essential at these stages to secure understanding of the value of each digit in a number e.g. place value counters, money etc. Wherever possible, decimal calculations should be linked to real life experiences.

## Development of Mental Strategies

The development of mental strategies is an important part of calculation and we aim for all children to be fluent in mental calculation, especially with regards to number bonds and multiplication tables relevant to their age and stage, in the following aspects:

### Addition Mental Calculations

#### EYFS

- Develop a mental image of the number system.
- Understand the value of a number
- Counting forwards and backwards
- Recall of number bonds to 10

#### Year 1

- Know addition can be carried out in any order (commutative)
- Add 1 and 2 digit numbers to 20 including 0
- Number bonds to 20
- Doubles of numbers up to and including double 10
- Adding 10 to a single digit number
- Identify 1 more than a given number

#### Year 2

- Know that addition is the inverse of subtraction  
Add numbers mentally, including:
  - ♣ A 2-digit number and units
  - ♣ A multiple of 10 to a 2-digit number
  - ♣ Two 2-digit numbers
  - ♣ Three 1-digit numbers
- Use knowledge of inverse to check calculations and solve missing number problems – Use knowledge of number bonds to 10 to calculate numbers bonds to 100 – Count on in tens from any given number (e.g 19 – 29 – 39 – 49 etc)

#### Year 3

- Add numbers mentally, including:
  - ♣ a three-digit number and a single digit number
  - ♣ a 3-digit number and multiples of 10
  - ♣ a 3-digit number and multiples of 100
- Estimate the answer to a calculation and use inverse operations to check answers
- Know number pairs that total 1000 (multiples of 100)
- Calculate 10 or 100 more than any given number

#### Year 4

- Add numbers mentally, including:
  - ♣ a four digit number and multiples of one thousand
- Use knowledge of doubles to derive related facts (e.g  $15 + 16 = 31$  because  $15 + 15 = 30$  and  $30 + 1 = 31$ )
- Know number pairs that total 1000 (multiples of 10)
- Estimate the answer to a calculation and use inverse operations to check answers

## Year 5

- Add numbers mentally with increasingly large numbers (e.g  $10,162 + 2,300 = 12,462$ )
- Mentally add tenths (e.g  $0.2 + 0.6 = 0.8$ ) and 1-digit whole numbers and tenths ( $8 + 0.3 = 8.3$ )
- Use number bonds to 100 knowledge to calculate complements to one using hundredths (e.g  $0.83 + 0.17 = 1$ )
- Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy

## Year 6

- Add numbers mentally with increasingly large numbers (e.g  $10,162 + 2,300 = 12,462$ )
- Add decimal numbers mentally (up to 2 decimal places)
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

## Subtraction Mental Calculations

## EYFS

- Develop a mental image of the number system
- Children count backwards using familiar number rhymes (e.g '10 Green Bottles', '5 Fat Sausages')
- Count backwards from different starting points

## Year 1

- Subtract 1 and 2 digit numbers to 20 including 0
- To know that subtraction is not commutative and that the larger number must always come first
- Use knowledge of number bonds to 10 and 20 to reason ( $9 + 1 = 10$  so  $10 - 9 = 1$  and  $10 - 1 = 9$ )

## Year 2

- To know that subtraction is the inverse of addition
- Use knowledge of inverse to check calculations and solve missing number problems
- Subtract numbers mentally, including:
  - ♣ subtracting units from a 2-digit number
  - ♣ subtracting a multiple of 10 from a 2-digit number
  - ♣ subtracting a 2-digit number from another 2-digit number
- Recall and use subtraction facts to 20 fluently
- Use knowledge of number bonds to 100 (multiples of 10) to reason ( $40 + 60 = 100$  so  $100 - 60 = 40$  and  $100 - 40 = 60$ )

## Year 3

- Subtract numbers mentally, including:
  - ♣ Subtracting a single digit number from a 3-digit number
  - ♣ Subtracting a multiple of 10 from a 3-digit number
  - ♣ Subtracting a multiple of 10 from a 3-digit number
- Estimate the answer to a calculation and use inverse operations to check answer

## Year 4

- Subtract numbers mentally, including: Subtracting multiples of one thousand from a 4-digit number
- Use of number pairs that total 1000 (multiples of 10) to calculate subtraction (e.g  $1000 - 300 = 700$ )
- Estimate the answer to a calculation and use inverse operations to check answers

## Year 5

- Subtract increasingly large numbers mentally (e.g  $12,654 - 1,341 = 11,213$ )
- Mentally subtract tenths (e.g  $0.7 - 0.5 = 0.2$ ) and 1-digit whole numbers and tenths ( $8 - 0.3 = 7.7$ )
- Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy

## Year 6

- Subtract increasingly large numbers mentally (e.g  $12,654 - 1,341 = 11,213$ )
- Subtract decimal numbers mentally (up to 2 decimal places)
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

## Multiplication Mental Calculations

### EYFS

- Develop a mental image of the number system.
- Understand the value of a number
- Counting in 2s, 5s and 10s.
- Number patterns on a number line and on a hundred square – 2's, 5's and 10's.

### Year 1

- Count forwards and backwards in multiples of 2s, 5s and 10s.
- Recall doubles of numbers up to and including 10.

### Year 2

- Count forwards and backwards in multiples of 3.
- Know the 2, 5 and 10 times tables (in and out of order)
- Recognise odd and even numbers

### Year 3

- Count forwards and backwards in multiples of 4, 8, 50 & 100
- Know the 3, 4 and 8 times tables (in and out of order)
- Connect the 2, 4 and 8 times tables through doubling
- Use knowledge of place value to calculate multiplication (e.g.  $2 \times 2 = 4$ ,  $2 \times 20 = 40$ ,  $2 \times 200 = 400$ )

### Year 4

- Know all times tables up to and including  $12 \times 12$  (by the end of Year 4)
- Recognise and use factor pairs (e.g factor pairs for numbers up to and including 10)
- Know that  $TU \times 5$  is  $TU \times 10$  then divide by 2 (e.g  $18 \times 5 = (18 \times 10) \div 2 = 90$ )
- Know that  $TU \times 9$  is  $TU \times 10$  then subtract  $TU$  (e.g  $18 \times 9 = (18 \times 10) - 18 = 162$ )

## Year 5

- Recognise and calculate factor pairs for any number
- Use times table knowledge to derive multiples of any number
- Establish whether a number is a prime number (up to 100) or a composite number (not prime) and recall prime numbers up to 19
- To know what a square number is and recall all square numbers (up to and including 144)
- To know what a cube number is and recall the first 5 cube numbers

## Year 6

- Use scaling to solve decimal number problems as whole number problems using the rule: 'the number of decimal digits in the question is the same as the number of decimal digits in the answer'
- Identify common factors, common multiples and prime numbers
- Use common factors to simplify fractions mentally
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

## Division Mental Calculations

### EYFS

- Develop a mental image of the number system.
- Understand the value of a number

### Year 1

- Count forwards and backwards in multiples of 2s, 5s and 10s

### Year 2

- To know that division is the inverse of multiplication
- Recall division facts for the 2, 5 and 10 times tables
- Recall halves for even numbers up to and including 20

### Year 3

- Know the division facts from the 3, 4 and 8 times tables
- Use knowledge of place value to calculate division (e.g.  $14 \div 2 = 7$ ,  $140 \div 2 = 70$ ,  $1400 \div 2 = 700$ )

### Year 4

- Know all related division facts for all times tables up to 12 times table (by the end of Year 4)

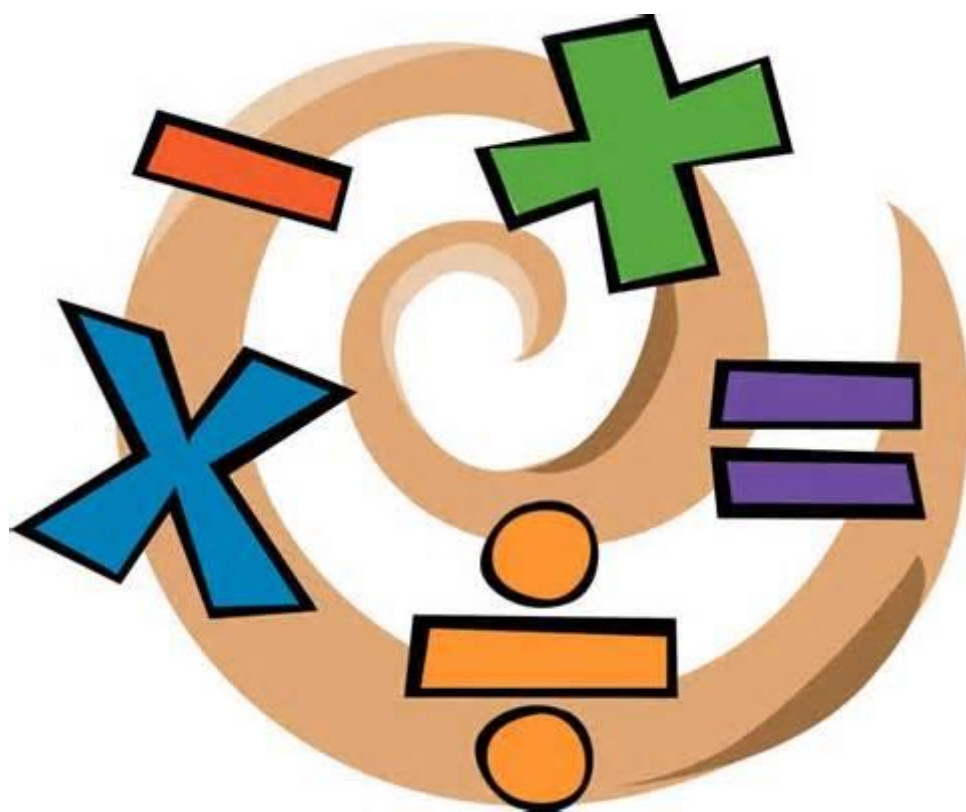
### Year 5

- Multiply and divide numbers mentally drawing upon known facts
- Associate fractions with division

### Year 6

- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy
- Calculate a fraction of an amount

# National Curriculum



# EYFS Early Learning Goals

# Statutory Requirements

## Mathematics - Number

DFE Statutory Framework for the Early Years Foundation Stage - April 2017, states that by the end of EYFS:

- Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number.
- Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.
- They solve problems, including doubling, halving and sharing.
- Children use everyday language to talk about time and money to compare objects and to solve problems.

The non-statutory 'Developmental Matters' Guidance for EYFS provides support for practitioners to implement the statutory requirements for EYFS. The main stages that we refer to for support in assessing children's development, under this guidance, are as follows:

### 30-50 months

- Uses some number names and number language spontaneously.
- Uses some number names accurately in play.
- Recites numbers in order to 10.
- Knows that numbers identify how many objects are in a set.
- Beginning to represent numbers using fingers, marks on paper or pictures.
- Sometimes matches numeral and quantity correctly.
- Shows curiosity about numbers by offering comments or asking questions.
- Compares two groups of objects, saying when they have the same number.
- Shows an interest in number problems.
- Separates a group of three or four objects in different ways, beginning to recognise that the total is still the same.
- Shows an interest in numerals in the environment.
- Shows an interest in representing numbers.
- Realises not only objects, but anything can be counted, including steps, claps or jumps.

### 40-60 months

Recognise some numerals of personal significance.

- Recognises numerals 1 to 5.
- Counts up to three or four objects by saying one number name for each item.
- Counts actions or objects which cannot be moved.
- Counts objects to 10, and beginning to count beyond 10.
- Counts out up to six objects from a larger group.
- Selects the correct numeral to represent 1 to 5, then 1 to 10 objects.
- Counts an irregular arrangement of up to ten objects.
- Estimates how many objects they can see and checks by counting them.
- Uses the language of 'more' and 'fewer' to compare two sets of objects.
- Finds the total number of items in two groups by counting all of them.
- Says the number that is one more than a given number.
- Finds one more or one less from a group of up to five objects, then ten objects.
- In practical activities and discussion, beginning to use the vocabulary involved in adding and subtracting.
- Records, using marks that they can interpret and explain.
- Begins to identify own mathematical problems based on own interests and fascinations.

*'Children develop at their own rates, and in their own ways. The development statements and their order should not be taken as necessary steps for individual children. They should not be used as checklists. The age/stage bands overlap because these are not fixed age boundaries but suggest a typical range of development.'*

*(Development Matters)*

## Year 1

- Count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number.
- Count, read and write numbers to 100 in numerals, count in multiples of 2, 5, and 10.
- Given a number, identify 1 more and 1 less
- Identify and represent numbers using objects and pictorial representations including the number line, and use the language of equal to, more than, less than, most, fewer and least.
- Read and write numbers from 1-20 in numerals and words.

## Year 2

- Count in steps of
  - 2, 3 and 5 from 0
  - 10 from any number
  - forwards and backwards
- Recognise the place value of each digit in a 2-digit number (tens, ones)
- Identify, represent and estimate numbers using different representations, including a number line.
- Compare and order numbers from 0 up to 100
  - using  $<$ ,  $>$  and  $=$  signs
- Read and write numbers to at least 100 in numerals and words
- Use place value and number facts to solve problems

## Year 3

- Count from 0 in multiples of 4, 8, 50 and 100
- Find 10 or 100 more or less than a given number
- Recognise the place value of each digit in a three-digit number (hundreds, tens, ones)
- Compare and order numbers up to 1000
- Identify, represent and estimate numbers using different representations
- Read and write numbers up to 1000 in numerals and in words
- Solve number problems and practical problems involving these ideas.

## Year 4

- Count in multiples of 6, 7, 9, 25 and 1000
- Find 1000 more or less than a given number
- Count backwards through zero to include negative numbers
- Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)
- Order and compare numbers beyond 1000
- Identify, represent and estimate numbers using different representations
- Round any number to the nearest 10, 100 or 1000
- Solve number and practical problems that involve all of the above and with increasingly larger
- Read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value

## Year 5

- Read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit
- Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000
- Interpret negative numbers in context
- Count forwards and backwards with positive and negative whole numbers, including through zero
- Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000
- Solve number problems and practical problems that involve all of the above
- Read Roman numerals to 1000 (M) and recognise years written in Roman numerals.

## Year 6

- Read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
- Round any whole number to a required degree of accuracy
- Use negative numbers in context, and calculate intervals across zero
- Solve number and practical problems that involve all of the above.

## Year 1

- Read, write and interpret mathematical statements involving mathematical signs addition + subtraction - equals =
- Represent and use number bonds and related subtraction facts within 20
- Add and subtract 1-digit and 2-digit numbers to 20, including 0
- Solve one step problems involving addition and subtraction using concrete objects and pictorial representation, using missing number problems e.g.  $7 = \square - 9$

## Year 2

- Solve problems with addition and subtraction
  - using concrete objects and pictorial representations
  - including those involving numbers, quantities and measures
- Recall and use addition and subtraction facts to 20 fluently
- Derive and use related facts up to 100
- Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and ones
  - a two-digit number and tens
  - two two-digit numbers
  - adding three one-digit numbers
- Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

## Year 3

- Add and subtract numbers mentally, including:
  - a three-digit number and ones
  - a three-digit number and tens
  - a three-digit number and hundreds
- Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
  - Estimate the answer to a calculation and use inverse operations to check answers
- Solve problems including missing number problems, using number facts, place value and more complex addition and subtraction.

## Year 4

- Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate (Mathematics Appendix 1)
- Estimate and use inverse operations to check answers to a calculation
- Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

## Year 5

- Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) (Mathematics Appendix 1)
- Add and subtract numbers mentally with increasingly large numbers
- Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

## Year 6 **Addition, Subtraction, Multiplication and Division**

- Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication (Mathematics Appendix 1)
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- Perform mental calculations, including with mixed operations and large numbers
- Identify common factors, common multiples and prime numbers
- Use their knowledge of the order of operations to carry out calculations involving the four operations
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Solve problems involving addition, subtraction, multiplication and division
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

## Year 1

- Solve one step problems involving multiplication and division, calculating the answer using concrete objects, pictorial representations and arrays With the support of the teacher

## Year 2

- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables
  - including recognising odd and even numbers
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using signs
  - multiplication  $\times$
  - division  $\div$
  - equals  $=$
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- Solve problems involving multiplication and division using
  - materials
  - arrays
  - repeated addition
  - mental methods
  - multiplication and division facts, including problems in contexts.

## Year 3

- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know including
  - -two-digit numbers times one-digit Numbers
    - using mental and progressing to formal written methods
- Solve problems including
  - missing number problems
  - involving multiplication and Division
  - positive integer scaling problems
  - correspondence problems in which  $n$  objects are connected to  $m$  objects

## Year 4

- Recall multiplication and division facts for multiplication tables up to  $12 \times 12$
- Use place value, known and derived facts to multiply and divide mentally, including:
  - multiplying by 0 and 1
  - dividing by 1
  - multiplying together three numbers
- Recognise and use factor pairs and commutativity in mental calculations
- Multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- Solve problems involving multiplying and adding, including:
  - using the distributive law to multiply two digit numbers by one digit
  - integer scaling problems
  - harder correspondence problems such as  $n$  objects are connected to  $m$  objects

## Year 5

- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- Establish whether a number up to 100 is prime and recall prime numbers up to 19
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- Multiply and divide numbers mentally drawing upon known facts
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- Recognise and use square numbers and cube numbers, and the notation for squared and cubed
- Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates

## Year 6

- See Previous section 'Addition, Subtraction, Multiplication and Division